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THE INFLUENCE OF PERMEABILITY AND OF A THIRD DIFFUSING COMPONENT UPON THE ONSET OF CONVECTION IN A POROUS MEDIUM

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Abstract—The linear stability of three-component system in a porous medium is investigated in the presence of a gravitationally stable density gradient. Particular attention is given to systems with $P_1 = 10^{-4}$, $\kappa_1 \gg \kappa_2$, κ_3 and $\nu \gg \kappa_2$. It is shown that the boundary for the onset of overstability can be approximated by two planar asymptotes in a Rayleigh number plane. Overstable and salt-finger modes are found to be simultaneously unstable when the density gradients due to the components are of the same sign and the effect of permeability of the porous medium is to suppress the regions of convection and salt-finger modes in the Rayleigh number plane.

NOMENCLATURE

a^2,	$= \pi^2(\alpha^2 + 1)$;
C_i,	concentration of the *i*th component [kg m^{-3}];
ΔC_i,	concentration difference between lower and upper layers [kg m^{-3}];
g,	acceleration due to gravity [m s^{-2}];
h,	a vertical length scale [m];
k,	permeability of porous medium [m^2];
p,	pressure of the system [kg m^{-1} s^{-2}];
$\mathbf{q}$,	$= (u,v,w)$, velocity vector [m s^{-1}];
$(x,y,z,)$,	Cartesian co-ordinates [m];
t,	time [s];
P_1,	$= k/h^2$, porous parameter;
P_1',	$= a^2 P_1$;
Pr,	$= \nu/\kappa_1$, the Prandtl or Schmidt number;
R_i,	$= gh^3\beta_i\Delta C_i/\nu\kappa_1$, Rayleigh number for the *i*th component.

Greek symbols

α,	the dimensionless wave number;
β_i,	expansion coefficients [m^3 kg^{-1}];
κ_i,	diffusivity of the *i*th component [m^2 s^{-1}];
ν,	kinematic viscosity of fluid [m^2 s^{-1}];
ρ,ρ_m,	density and mean density of fluid;
τ_i,	$= \kappa_i/\kappa_1$, ratio of diffusivities;
λ_i,	$= \pi^2\alpha^2 R_i/a^6$;
λ_s,	$= \lambda_2 + \lambda_3$, the salinity Rayleigh number;
ω,	$= \sigma/a^2$;
σ,	= frequency [s^{-1}].

1. INTRODUCTION

FOR MANY areas of technology such as chemical engineering, petroleum industries and geothermal activity the study of heat and mass transfer through a porous medium is of significance. This is usually influenced by free convection whereby the void fluid comes into circulation as a result of density differences caused by temperature or concentration. Therefore, there has been considerable interest recently in the study of convection through a porous medium (see [1]). The available literature on the study of free convection through a porous medium is usually limited by a single component fluid in which there is no stabilizing or destabilizing gradient of concentration.

Free convection of a two-component fluid saturated porous layer driven by the differential diffusion of two properties such as heat and salt is also of interest in numerous practical fields, particularly in some oil recovery techniques. Although copious literature on this dealing with both linear and non-linear theories is available in the case of pure viscous flow (see [2]), very little is known of the case of a fluid saturated porous layer. Nield [3] has studied the convection of a two-component fluid saturated porous layer using infinitesimal amplitude analysis by considering two different diffusive properties, one of which is stabilizing and the other destabilizing. Here the emphasis is put on the boundary conditions rather than determining analytical expressions for the onset of convection and heat transport.

Rudraiah and Prabhamani [4, 5] have studied the linear and non-linear stability of a two-component fluid in a porous medium in the presence of thermal diffusion (i.e. Soret effect) using the Liapunov technique. Their analysis gives only the condition for the onset of convection but says nothing about the prediction of heat transport. Recently Rudraiah *et al.* [6] and Srimani [7] have made a detailed analysis of convection in a two-component fluid saturated porous layer, using the local non-linear stability analysis, where the condition for onset of convection and its effect on heat transfer are reported.

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The literature cited above concentrates only on doubly diffusive convection in a fluid saturated porous medium. There are many fluid saturated porous systems, however, in which more than two components are present. For example, in geothermal regions the lower part of the earth's crust, particularly the aquifers, is considered to be a fluid saturated porous medium consisting of multi-components of fluid. An extensive body of chemical and physical data of multi-component fluid saturated porous medium in geothermal regions now exists and a proper theory has to be developed on the basis of this data. Therefore, a proper understanding of field behaviour in geothermal regions should include convection of a multi-component fluid saturated porous medium.

In the usual viscous flow, Griffiths [8, 9] has considered the influence of a third diffusive component upon the onset of convection and has shown that oscillatory and direct salt-finger modes are simultaneously unstable under a wide range of conditions when the density gradients due to the components with the greatest and smallest difiusivities are of the same sign. For geophysical applications mentioned above, we have to take into account the effect of resistance offered by the solid particles to the fluid because the aquifers are considered to flow through porous media. Therefore, the present analysis was undertaken to provide results of a small amplitude stability analysis for linear concentration gradients of three components in a porous medium following the analysis of Griffiths [8].

2. MATHEMATICAL FORMULATION

Consider a fluid saturated porous layer between two horizontal parallel stress free boundaries at $z = 0$ and h and containing three diffusing properties such that

$$\kappa_1 > \kappa_2 > \kappa_3 \tag{2.1}$$

where $\kappa_i (i = 1, 2, 3)$ is the coefficient of molecular diffusion for the ith component. The boundaries are perfectly conducting for solute and the z axis is chosen to be directed vertically upwards and the basic property gradients are assumed to be vertical and linear. The concentration difference between the boundaries is ΔC_i with the sign convention that $\Delta C_i > 0$, when the component is destabilizing. With the assumptions and approximations which are frequently used for convection in a homogeneous porous medium saturated with three component Boussinesq fluid, the general equations may be written as

$$\frac{\partial \mathbf{q}}{\partial t} = -\frac{1}{\rho_m}\nabla p - \frac{\nu}{k}\mathbf{q} + \frac{\rho}{\rho_m}\mathbf{g} \tag{2.2}$$

$$\left(\frac{\partial}{\partial t} - \kappa_i \nabla^2\right) C_i = -\nabla \cdot (\mathbf{q} C_i) \tag{2.3}$$

$$\nabla \cdot \mathbf{q} = 0 \tag{2.4}$$

$$\rho = \rho_m [1 + \sum_i \beta_i C_i]. \tag{2.5}$$

In order to carry out the analysis it is convenient to non-dimensionalize equations (2.2–2.4) scaling with h and κ_i. We then have Rayleigh numbers R_i, diffusivity ratios τ_i, Prandtl number Pr and a porous parameter P_1 given by

$$R_i = \frac{g h^3 \beta_i \Delta C_i}{\nu \kappa_1}, \quad \tau_i = \frac{\kappa_i}{\kappa_1}, \quad Pr = \frac{\nu}{\kappa_1},$$
$$P_1 = k/h^2. \qquad (i = 1,2,3) \tag{2.6}$$

In this paper, we consider only two-dimensional motions with velocity $\mathbf{q} = (u,0,w)$ and the stream function ψ as

$$u = \frac{\partial \psi}{\partial z}, \quad w = -\frac{\partial \psi}{\partial x}. \tag{2.7}$$

Then the linearized perturbation equations (2.2–2.4) after eliminating the pressure are

$$\left(\frac{1}{Pr}\frac{\partial}{\partial t} + \frac{1}{P_1}\right)\nabla^2 \psi = \sum_{i=1}^{3} \frac{\partial \theta_i}{\partial x}, \tag{2.8}$$

$$\left(\frac{\partial}{\partial t} - \tau_i \nabla^2\right)\theta_i = R_i \frac{\partial \psi}{\partial x} \tag{2.9}$$

where the θ_i are non-dimensional concentrations. The boundary conditions are

$$\theta_i = \psi = \frac{\partial^2 \psi}{\partial z^2} = 0, \quad \text{at } z = 0 \text{ and } 1. \tag{2.10}$$

3. LINEAR STABILITY ANALYSIS

Eliminating θ_i in equations (2.8) and (2.9) and assuming the normal mode solutions of the form $e^{\sigma t} \cos \pi \alpha x \sin \pi z$ with $\sigma = \sigma_r + i\sigma_i$ we obtain the dispersion relation

$$\frac{\omega^4}{Pr} + \frac{\omega^3}{Pr}\left(\frac{Pr}{P_1'} + 1 + \tau_2 + \tau_3\right) + \omega^2\left[\frac{1 + \tau_2 + \tau_3}{P_1'} + \frac{\tau_2}{Pr} + \frac{\tau_3}{Pr} + \frac{\tau_2\tau_3}{Pr} - \lambda_1 - \lambda_2 - \lambda_3\right] + \omega\left[\frac{\tau_2 + \tau_2\tau_3 + \tau_3}{P_1'} + \frac{\tau_2\tau_3}{Pr} - (\tau_2 + \tau_3)\lambda_1 - (1 + \tau_3)\lambda_2 - (1 + \tau_2)\lambda_3\right] + \tau_2\tau_3\left(\frac{1}{P_1'} - \lambda_1 - \frac{\lambda_2}{\tau_2} - \frac{\lambda_3}{\tau_3}\right) = 0 \tag{3.1}$$

with $\omega = \sigma/a2$, $P_1' = a^2 P_1$, $\lambda_2 = (\pi^2\alpha^2/a^6)\, R_i$ and $a^2 = \pi^2(\alpha^2 + 1)$.

We note that this dispersion relation (3.1) is of 4th-order in contrast to the cubic equation obtained by Rudraiah *et al.* [6] in the case of two-component fluid. We also note that when $P_1' \to 1$ (i.e. $P_1 = 1/a^2$) this dispersion relation tends to the one given by Griffiths [8] in the case of viscous flow. In other words when $P_1 = \pi^{-2}(\alpha^2 + 1)^{-1}$ we can get the stability results of viscous flow discussed by [8]. This surprising result may be due to the fact that although the order of the derivatives in Darcy and viscous flow are different the order of the time derivatives remain the same. This result is purely local and not global.

3.1. *Marginal state*

Marginal state is valid when $\sigma_r = 0$ and $\sigma_i = 0$. For the most unstable mode at marginal stability

$$\alpha^2 = 1, \quad \lambda_i = \frac{1}{8\pi^4} R_i. \tag{3.2}$$

Relations (3.1) and (3.2) imply that for the most unstable mode ω is real and positive in the half space

$$\sum_{i=1}^{3} \frac{R_i}{\tau_i} \geqslant \frac{4\pi^2}{P_1} \tag{3.3}$$

so that instability is monotonic there. Equation (3.1) together with (3.2) reveals that one of its roots is zero, which may not always be the root of maximum growth rate when the equality holds in (3.3). In other words, the principal of exchange of stability is valid when

$$\sum_{i=1}^{3} \frac{R_i}{\tau_i} = \frac{4\pi^2}{P_1} \tag{3.4}$$

if the density gradient is gravitationally stable. Inequality (3.3) is the generalization of Lapwood [10] criterion

$$R_1 \geqslant 4\pi^2/P_1$$

for convection in a single component fluid saturated porous medium. The plane surface of marginal stability on which the relation (3.4) holds will be denoted by $\mathscr{P}$.

3.2. *Overstable state*

In a three-component fluid saturated porous medium discussed here, the applied concentration gradient makes the velocity and concentration distributions out of phase and hence an overstable system or oscillatory system exists [11] for certain values of R_i. For overstable mode $\sigma_r = 0$ and $\sigma_i \neq 0$ that is, $\sigma = i\sigma_i$ where σ_i has to be real. Substituting $\omega = i\omega_i$ in (3.1) and equating the real and imaginary parts to zero we have

$$\frac{\omega_i^4}{Pr} - \omega_i^2 (D - \lambda_1 - \lambda_2 - \lambda_3) + \tau_2\tau_3 \times \left(\frac{1}{P_1'} - \lambda_1 - \frac{\lambda_2}{\tau_2} - \frac{\lambda_3}{\tau_3}\right) = 0, \tag{3.5}$$

$$\omega_i^2 = \left(\frac{Pr}{B}\right)\left[A - (\tau_2 + \tau_3)\lambda_1 - (1 + \tau_3)\lambda_2 - (1 + \tau_2)\lambda_3\right] \tag{3.6}$$

provided $\omega_i \neq 0$ where

$$A = \frac{\tau_2 + \tau_3}{P_1'} + \tau_2\tau_3\left(\frac{1}{P_1'} + \frac{1}{Pr}\right),$$

$$B = \frac{Pr}{P_1'} + 1 + \tau_2 + \tau_3,$$

$$D = \frac{1}{P_1'} + \frac{\tau_2 + \tau_3}{P_1'} + \frac{1}{Pr}(\tau_2 + \tau_3 + \tau_2\tau_3).$$

Equation (3.5) together with (3.2) and (3.6) describes a conicoid

$$a_1 R_1^2 + a_2 R_2^2 + a_3 R_3^2 + a_4 R_1 R_2 \; a_5 R_1 R_3 + a_6 R_2 R_3 + 8\pi^4 \times (a_7 R_1 + a_8 R_2 + a_9 R_3) + (8\pi^4)^2 a_0 = 0 \tag{3.7}$$

where

$$a_1 = -\left(1 + \frac{Pr}{P_1'}\right)(\tau_2 + \tau_3),$$

$$a_2 = -\left(\tau_2 + \frac{Pr}{P_1'}\right)(1 + \tau_3),$$

$$a_3 = -\left(\tau_3 + \frac{Pr}{P_1'}\right)(1 + \tau_2),$$

$$a_4 = 2(1 + \tau_3)(\tau_2 + \tau_3) - B(1 + 2\tau_3 + \tau_2),$$

$$a_5 = 2(1 + \tau_2)(\tau_2 + \tau_3) - B(1 + 2\tau_2 + \tau_3),$$

$$a_6 = 2(1 + \tau_3)(1 + \tau_2) - B(2 + \tau_2 + \tau_3),$$

$$a_7 = (BD - 2A)(\tau_2 + \tau_3) + AB - \tau_2\tau_3 B^2/Pr,$$

$$a_8 = (BD - 2A)(1 + \tau_3) + AB - \tau_3 B^2/Pr,$$

$$a_9 = (BD - 2A)(1 + \tau_2) + AB - \tau_2 B^2/Pr,$$

$$a_0 = A^2 - ABD + \tau_2\tau_3 B^2/Pr\, P_1'.$$

For overstable motion we require $\omega_i^2 > 0$ in (3.5) and (3.6). This is possible only if the inequalities

$$(\tau_2 + \tau_3)R_1 + (1 + \tau_3)R_2 + (1 + \tau_2)R_3 \leqslant 8\pi^4 A, \tag{3.8}$$

$$\frac{8\pi^4}{P_1'} \leqslant R_1 + \frac{R_2}{\tau_2} + \frac{R_3}{\tau_3} \tag{3.9}$$

and

$$8\pi^4 D - R_1 - R_2 - R_3 \pm G^{1/2} \geqslant 0 \tag{3.10}$$

are satisfied on (3.7) where

$$G = R_1^2 + R_2^2 + R_3^2 + 2(R_1R_2 + R_1R_3 + R_2R_3) + 8\pi^4 \left[4\tau_2\tau_3 Pr^{-1}(R_1 + R_2\tau_2^{-1} + R_3\tau_3^{-1}) - 2D(R_1 + R_2 + R_3)\right] + (8\pi^4)^2\left(D^2 - 4\tau_2\tau_3\frac{Pr^{-1}}{P_1'}\right).$$

Equation (3.7) together with conditions (3.8–3.10) represent different surfaces depending on the values of τ_i. For example, for $0 < \tau_2$ and $\tau_3 < 1$, (3.7) represents an hyperboloid denoted by $\mathscr{H}$, of which inequalities (3.8–3.10) select the appropriate branch giving rise to the conditions at which the most unstable oscillatory mode is marginally stable. On the other hand, in the case of single component fluid that is $\tau_2 = \tau_3 = 1$, (3.7) represents a paraboloid. Further the interesection of (3.7) with any plane parallel to the plane $R_1 = 0$ becomes parabolic when $\tau_2, \tau_3 = 0$.

We note that the complex roots of the quartic characteristic equation (3.1) may become real roots without their parts passing through zero and hence the plane boundary $\mathscr{P}$ and the hyperboloid $\mathscr{H}$ alone are not sufficient to determine fully the conditions under which marginal or oscillatory stabilities exist. Two such transitions, as in the case of three-component pure viscous flow discussed by [8] occur for the quartic equation (3.1) discussed here. The possible positions of the boundaries $\mathscr{P}$ and $\mathscr{H}$ are illustrated in the next section for two sets of molecular properties.

4. EXAMPLES OF STABILITY BOUNDARIES

In this section the stability boundaries are discussed by taking specific examples based on the values of τ_2 and τ_3.

4.1. *A case where* $\tau_2, \tau_3 \leqslant 1$

Since the aim of this paper is to understand the mechanism of convection in geothermal regions, we consider the values of τ_2 and τ_3 pertaining to this region. It is believed that the fluid in a geothermal region consists of aqueous solution of KCl, NaCl and sucrose having the values of molecular diffusion (see [8])

$$\kappa_1 = 1.6 \times 10^{-5}\,\mathrm{cm^2\,s^{-1}}, \quad \kappa_2 = 1.3 \times 10^{-5}\,\mathrm{cm^2\,s^{-1}},$$
$$\kappa_3 = 0.45 \times 10^{-5}\,\mathrm{cm^2\,s^{-1}}, \quad P_1 = 10^{-4} \tag{4.1}$$

and Prandtl number $Pr = 625$. These values are also typical of laboratory models using salt–sugar solution where the diffusivity rates are not far from unity.

Choosing specific values of P_1, the relevant portions of the intersections of $\mathscr{P}$ and $\mathscr{H}$ with the R_3 plane are computed. By relevant portions we mean those which describe a change in the mode of instability for the most unstable mode. Figures 1(a–c) show the relevant portions of the intersections of $\mathscr{P}$ and $\mathscr{H}$ with three planes $\lambda_3 = -50$, $\lambda_3 = 0$ and $\lambda_3 = 50$. The plane $\mathscr{S}$ defined by

$$\sum_i R_i = 0 \tag{4.2}$$

and on which $\partial\rho/\partial z = 0$ is also shown.

The asymptotes of the hyperbola are almost parallel, the slope of each being independent of the value of λ_3. The upper asymptote has a slope

$$-\left(\frac{Pr}{P_1'} + \tau_2\right)\Big/\left(\frac{Pr}{P_1'} + 1\right) \quad (\approx 1).$$

In Figs. 1(a–c), the oblique and horizontal hatchings denote respectively the existence of regions of salt fingers and overstable modes when the fastest and slowest diffusing components are destabilizing for example, the second quadrant of Fig. 1(c) when λ_3 is sufficiently large and more extensively when these components are stabilizing for example, the fourth quadrant of Fig. 1(a).

4.2. *A case where* $\tau_2, \tau_3 \ll 1$

As in the case of viscous flow [8], the discussion of three component system in a porous medium may be simplified if $\tau_2, \tau_3 \to 0$ and $Pr \gg \tau_2$ for $\mathscr{H}$ approaches its asymptotes in this limit and the asymptotes themselves degenerate into a pair of planes:

$$\lambda_1 + \frac{Pr/P_1'}{\left(\frac{Pr}{P_1'} + 1\right)}(\lambda_2 + \lambda_3) = \frac{1}{P_1'}, \tag{4.3}$$

$$\lambda_1 + (\tau_2 + \tau_3)^{-1}(\lambda_2 + \lambda_3) = \frac{1}{P_1'}. \tag{4.4}$$

When $\lambda_3 = 0$, relation (4.3) reduces to the condition for marginal stability of oscillatory modes of two-component systems investigated by Rudraiah *et al.* [6].

As a specific example we consider the aqueous system heat–KCl–sucrose ($i = 1,2,3$ respectively) for which (Griffiths [8]) $\kappa_1 = 1.4 \times 10^{-3}\,\mathrm{cm^2 s^{-1}}$, $\kappa_2 = 1.6 \times 10^{-5}\,\mathrm{cm^2 s^{-2}}$, $\kappa_3 = 0.45 \times 10^{-5}\,\mathrm{cm^2\,s^{-1}}$ and $Pr = 7$. Figures 2(a–c) show the relevant portions of the intersections of $\mathscr{P}$, $\mathscr{H}$ and $\mathscr{S}$ with the three planes $\lambda_3 = -350$, $\lambda_3 = 0$ and $\lambda_3 = 350$ respectively. It is easy to see that $\mathscr{H}$ can be described very closely by (4.3) and (4.4).

To depict the 3-dim. geometry more clearly, the intersection of the surfaces $\mathscr{P}$, $\mathscr{H}$ and $\mathscr{S}$ with each other, as functions of λ_3 will also be defined and are shown as broken lines in Fig. 2. The two asymptotes (4.3) and (4.4) intersect at the point A where $\lambda_1 = 1/P_1'$ and $\lambda_2 = -\lambda_3$. This point lies to the left of $\mathscr{P}$ when $\lambda_3 < 0$. The lower asymptote (4.4) of $\mathscr{H}$ and the plane $\mathscr{P}$:

$$\lambda_1 + \frac{\lambda_2}{\tau_2} + \frac{\lambda_3}{\tau_3} = \frac{1}{P_1'}$$

converge slowly to intersect at a point P where

$$\lambda_1 = \frac{1}{P_1'} - |\lambda_3|\frac{1 - \tau_*}{\tau_* \tau_3}, \tag{4.5}$$

$$\lambda_2 = \frac{|\lambda_3|}{\tau_*^2} \tag{4.6}$$

where

$$\tau_* = \tau_3/\tau_2 = \kappa_3/\kappa_2.$$

The locus of P for different values of λ_3 is computed and is shown in the fourth quadrant of Fig. 2(a) as the almost vertical broken line. The planes $\mathscr{P}$ and $\mathscr{S}$ converge slowly to interact at the point B where

$$\lambda_1 = \frac{1}{(1 - \tau_2)}\left[\frac{\lambda_3(1 - \tau_*)}{\tau_*} - \frac{\tau_2}{P_1'}\right], \tag{4.7}$$

$$\lambda_2 = \frac{\tau_2}{(1 - \tau_2)P_1'} - \frac{\lambda_3(1 - \tau_3)}{(1 - \tau_2)\tau_*}. \tag{4.8}$$

The point P falls below B. Further, the point P lies within the region of static stability, so that a stabilizing gradient of the component with smallest diffusivity generates a range of values of λ_2 and λ_3 at which overstable modes occur.

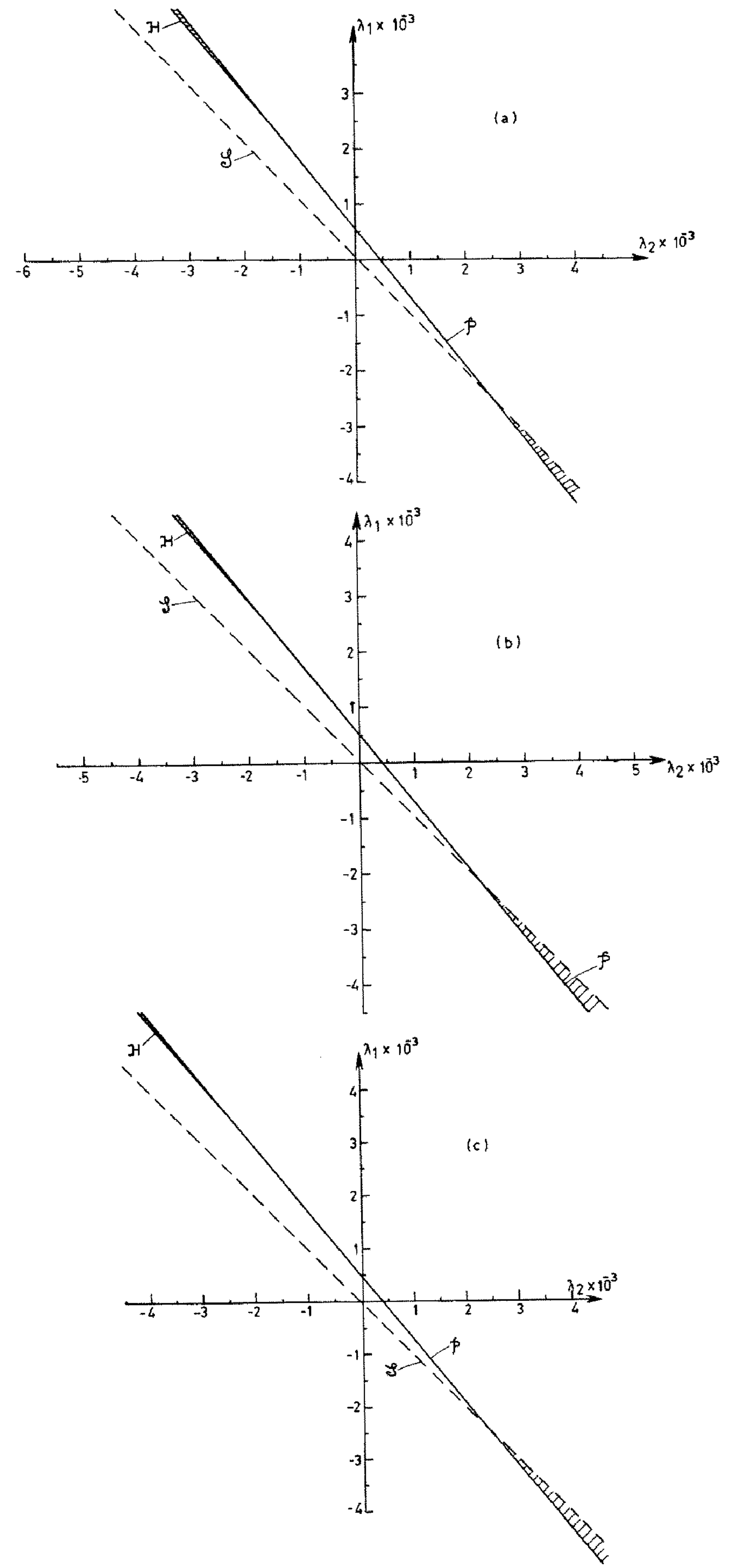
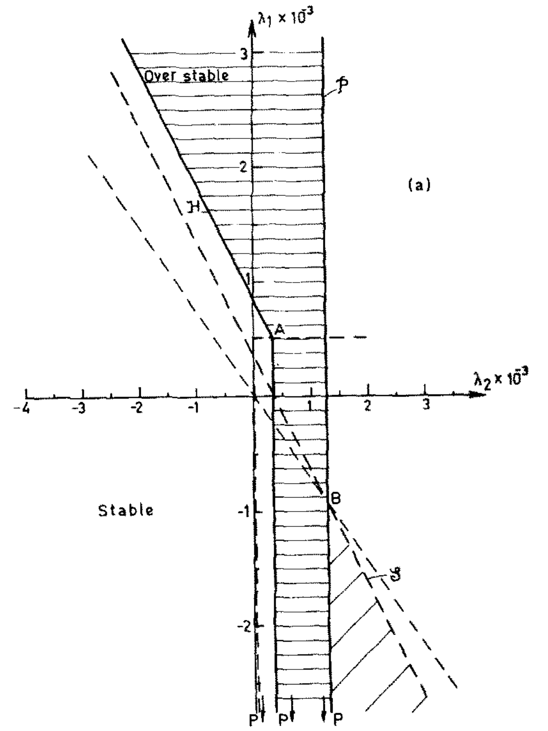
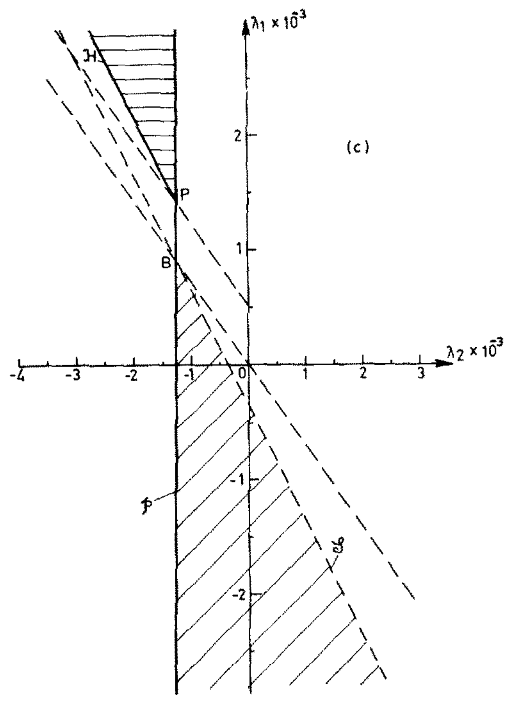


FIG. 1(a), (b). Stability boundaries for the most unstable mode when $\tau_2 = 0.81$, $\tau_3 = 0.28$, $\tau_* = 0.35$, $Pr = 625$ and $P_1 = 10^{-4}$ at three values of λ_3: (a) $\lambda_3 = -50$, (b) $\lambda_3 = 0$, (c) $\lambda_3 = 50$. The coordinates are normalized with respect to the critical Rayleigh number $8\pi^4$ and the lines are explained in the text. Horizontally hatched regions give overstable modes; oblique hatching shows conditions unstable to salt fingers.





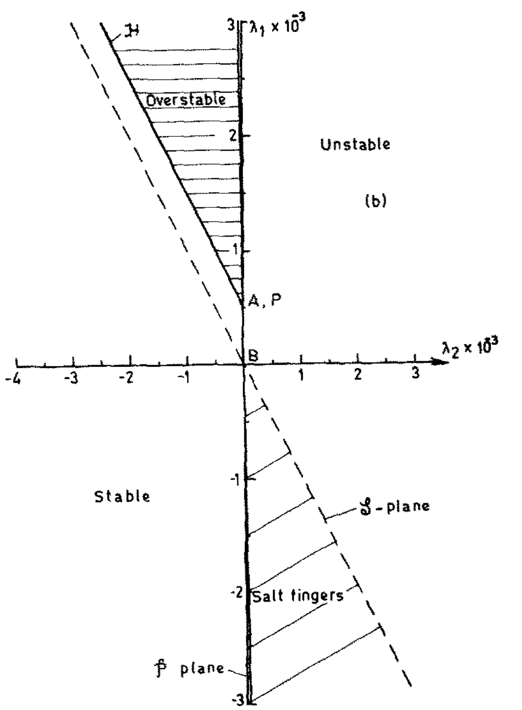


FIG. 2(a), (b). Stability boundaries for the most unstable mode when $\tau_2 = 0.011$, $\tau_3 = 0.003$, $\tau_* = 0.28$, $Pr = 7$ and $P_1 = 10^{-4}$ at three values of λ_3: (a) $\lambda_3 = -350$, (b) $\lambda_3 = 0$, (c) $\lambda_3 = 350$. Hatching and heavy lines have the same meaning as in Fig. 1 and the points A, B and P move along the fine broken lines when λ_3 is varied.

For $\lambda_3 > 0$ (see Fig. 2c) the point A lies on the right of $\mathscr{P}$ so that only one asymptote (4.4) is drawn. The intersection $\mathscr{P}$ now lies on the line

$$\lambda_1 + \frac{Pr(1-\tau_*)\lambda_2}{(Pr+P_1')} = \frac{1}{P_1'} \tag{4.9}$$

in the second quadrant of the λ_2, λ_1 plane. We also note that (4.3) and $\mathscr{S}$ intersect at the point where

$$\lambda_1 = \frac{Pr+P_1'}{(P_1')^2}, \tag{4.10}$$

$$\lambda_2 = \frac{-(Pr+P_1')}{(P_1')^2} - \lambda_3. \tag{4.11}$$

For $\lambda_3 > (Pr + P_1')\tau_*/[P_1'(1-\tau_*)]$ (≈ 1379.5 in the present example), there are two points of intersection with $\mathscr{P}$ below B. We note that a destabilizing gradient of component 2 ($\lambda_2 > 0$) is no longer a necessary condition for the growth of salt-fingers.

This stability analysis is concerned only with the critical wave number given by equation (3.2). However, if we allow the wave number α to vary, this stability analysis yields further information on the physical behaviour of multi-component systems in a porous medium as explained below.

When we allow α to vary, the intersection $\mathscr{P}$ in Fig. 2 must now divide conditions to the left, where only overstable motions are unstable, from conditions to the right where both overstable and marginal modes are possible. This behaviour is illustrated, following

Griffiths [8], schematically on the R_2R_1 plane in Fig. 3(a) for $R_3 = -\zeta < 0$ and in Fig. 3(b) for $R_3 = \eta > 0$. As R_3 is varied the intersections A, B and P move along the fine broken lines. Further, the values of R_2 at the intersections of $\mathscr{S}$, $\mathscr{P}$ and $\mathscr{L}$ with the R_2 axis are also marked.

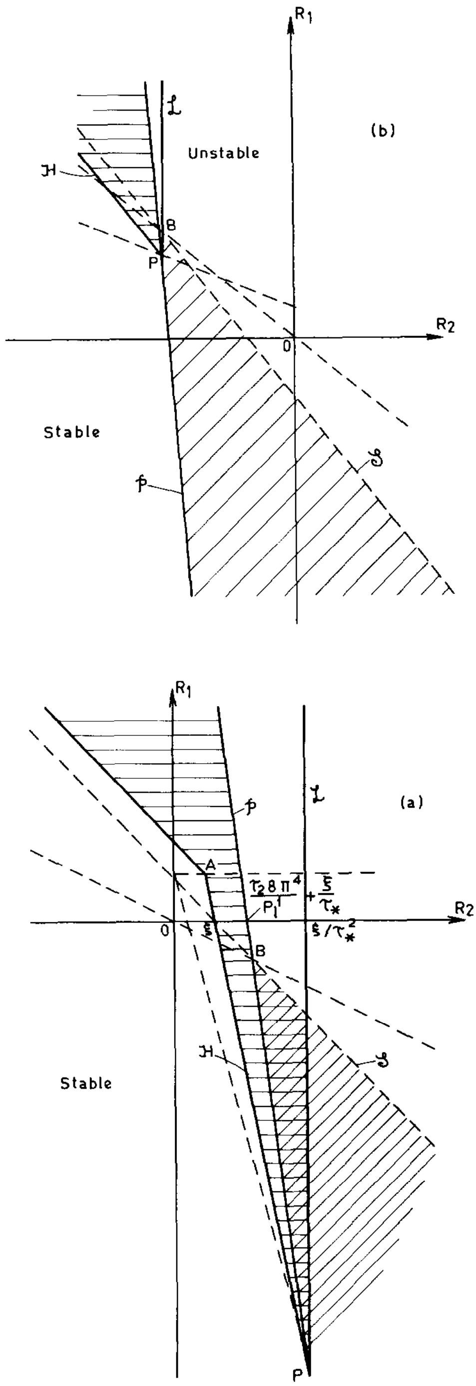


FIG. 3 (a). Schematic three-component stability bounds (heavy lines) for (a) $R_3 = -\zeta < 0$, stabilising and (b) $R_3 = -\eta > 0$, destabilising. Values of R_2 at intersections with $R_1 = 0$ are marked. Both oscillatory and salt-finger modes are unstable in the double hatched region.

In Fig. 3, another plane surface $\mathscr{L}$ is drawn such that to the right of which no wavelengths are overstable and only monotonic instability is possible. This plane $\mathscr{L}$ is the locus of the intersections of $\mathscr{H}$ and $\mathscr{P}$ as the term $\pi^2\alpha^2/k^6$ is allowed to vary in (3.7). In the limit $\tau_2, \tau_3 \to 0$, (3.1), (3.3), (4.3) and (4.4) imply that $\mathscr{L}$ is vertical, while for the two-component case and arbitrary molecular properties, Rudraiah *et al.* [6] have shown that the appropriate line can be obtained from

$$R_1 = \frac{(\tau_2 + Pr/P_1')R_2}{(1 + Pr/P_1')} + (1 + \tau_2)\left(1 + \frac{\tau_2}{Pr/P_1'}\right)\frac{8\pi^4}{P_1'}.$$

To understand more clearly, consider a system which lies in the fourth quadrant of Fig. 3(a) and which always has $d\rho/dz < 0$. Then, for a given P_1, the components with the greatest and smallest diffusivities, κ_1 and κ_3, both contribute negative density gradients opposing that due to component 2. When R_2 and P_1' are sufficiently small, the system is stable. However, if R_2, for a given P_1' is increased until the (R_1, R_2, R_3) coordinates cross $\mathscr{H}$ when some wavelengths become overstable, beginning at $\alpha = 1$, the mode which represents a balance between more efficient diffusive transport and Darcy damping. If R_2 is more stabilizing so that the conditions are just to the right of the plane $\mathscr{P}$, the mode with wavenumber $\alpha = 1$ grows monotonically, with other modes remaining overstable. R_2 is now large enough to overcome the stable stratification as well as Darcy resistance and at larger values, to the right of $\mathscr{L}$, all unstable modes are direct.

5. CONCLUSIONS

The stability analysis of a three-component system in a porous medium, investigated using infinitesimal disturbances, reveals that: (i) the marginal stability of oscillatory modes occurs on a hyperboloid in Rayleigh number space but the surface is very closely approximated by its planar asymptotes for any diffusivity ratios. The effect of permeability is to reduce the region of salt-finger and overstable modes, and (ii) when the density gradients due to the components with the greatest and smallest diffusivities are of the same sign, salt-finger and overstable modes may be simultaneously unstable over a wide range of conditions and the effect of permeability is to suppress these modes.

The stability analysis reveals that small concentrations of slowly diffusing properties are important because the influence of a component upon marginally stable oscillatory modes is proportional to $\beta_i(\Delta C_i)/P_1'$ while the influence of any component upon the occurrence of salt-fingers is proportional to $|\beta_i \Delta C_i|/P_1'\kappa_i$. For cases in which $\tau_2, \tau_3 \ll 1$, the conditions (4.3) and (4.4) for neutral stability of overstable modes can be written in terms of a total salinity Rayleigh number $\lambda_s = \lambda_2 + \lambda_3$.

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INFLUENCE DE LA PERMEABILITE ET D'UN TROISIEME COMPOSANT DIFFUSANT, SUR LE DEBUT DE LA CONVECTION DANS UN MILIEU POREUX

Résumé—On étudie la stabilité linéaire d'un système à trois composants dans un milieu poreux, en présence d'un gradient de densité gravitationnellement stable. Une attention particulière est portée sur les systèmes avec $Pe = 10^{-4}$, $K_1 \gg K_2, K_3$ et $\nu \gg K_2$. On montre que la frontière pour le déclenchement de la surstabilité peut être approchée par deux asymptotes dans un plan de nombres de Rayleigh. Les modes surstables et à digitation sont simultanément surstables quand les gradients de densité dûs aux composants sont de même signe et l'effet de la perméabilité du milieu poreux est de supprimer les régions de modes de convection et de digitation dans le plan de nombres de Rayleigh.

DER EINFLUSS DER PERMEABILITÄT UND EINER DRITTEN DIFFUNDIERENDEN KOMPONENTE AUF DAS EINSETZEN DER KONVEKTION IN EINEM PORÖSEN MEDIUM

Zusammenfassung—Die lineare Stabilität eines Dreikomponenten-Systems in einem porösen Medium wird für den Fall eines gravitationsbedingt stabilen Dichtegradienten untersucht. Das besondere Interesse gilt dabei Systemen mit $P_l = 10^{-4}$, $K_1 \gg K_2, K_3$ and $\nu \gg K_2$. Es wird gezeigt, daß die Grenze für das Auftreten von Überstabilität durch zwei ebene Asymptoten in einer Rayleigh-Zahl-Ebene angenähert werden kann. Überstabilitäts- und "Salzfinger"-Gebiete erweisen sich als gleichzeitig überstabil, wenn die durch die Komponenten bedingten Dichtegradienten dasselbe Vorzeichen haben und der Einfluß der Permeabilität des porösen Mediums den Bereichen der Konvektions- und der "Salzfinger"-Gebiete in der Rayleigh-Zahl-Ebene entgegenwirkt.

ВЛИЯНИЕ ПРОНИЦАЕМОСТИ И ТРЕТЬЕГО ДИФФУНДИРУЮЩЕГО КОМПОНЕНТА НА ВОЗНИКНОВЕНИЕ КОНВЕКЦИИ В ПОРИСТОЙ СРЕДЕ

Аннотация — Исследуется линейная устойчивость трехкомпонентной системы в пористой среде при наличии градиента плотности, не зависящего от действия силы тяжести. Особое внимание обращено на системы с $P_l = 10^{-4}$, $k_1 \gg k_2$, k_3 и $\nu \gg k_2$. Показано, что границу возникновения «сверхустойчивости» можно аппроксимировать двумя плоскими асимптотами в плоскости чисел Релея. Найдено, что «сверхустойчивый» режим и режим типа «соляного пальца» наблюдаются одновременно в том случае, когда градиенты плотности компонентов системы имеют тот же знак, а проницаемость пористой среды приводит к подавлению конвективного режима и режима «соляного пальца» в плоскости чисел Релея.